QP CODE: 26017606

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Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE / MERCY CHANCE EXAMINATIONS, MARCH 2026

Sixth Semester

CORE COURSE - MM6CRT03 - COMPLEX ANALYSIS

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

6DC8EAB0

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find the imaginary part of the function f(z)=$\tan z$
2. Find f'(z) where f(z)=$\frac{z-1}{2z+1}$ where z$\neq \frac{-1}{2}$
3. Give an example of a function which is differentiable only at the origin.Illustrate.
4. Prove that real and imaginary part of an analytic function are harmonic.
5. Define the hyperbolic sine and hyperbolic cosine of a complex variable z
6. Evaluate $\int_1^2 (\frac{1}{t} - i)^2 dt.$
7. Evaluate $\int_{|z|<2} \frac{ze^z}{(z^2+9)^5} dz$
8. If C is the positive oriented circle |z|=1, find $\int_C \frac{dz}{z}$
9. *Evaluate* $lim_{n\to\infty} z_n$ *where* $z_n = \frac{-2+i(-1)^n}{n^2}$
10. *Obtain a Maclaurin series expansion of* z^2e^{3z}
11. Define residue of a complex function and state its use in contour integration.
12. Prove that if the improper integral over $-\infty < x < \infty$ exists, then its Cauchy Principal Value exists.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Prove that |exp(-2z)|<1 if and only if Re(z)>0

14. Show that $Re[\log(z-1)] = \frac{1}{2}\ln[(x-1)^2+y^2], z \neq 1$

15. Find where $\tan^{-1} z = \frac{i}{2} \log \frac{i+z}{i-z}$ is analytic

16. Evaluate $\int_C \frac{z+2}{z} dz$, where C is the circle $z = 2e^{i\theta}$, $(0 \leq \theta \leq 2\pi)$.

17. Let C be the arc of the circle |z|=2 from z=2 to z=2i that lies in the first quadrant. Then without evaluating the integral show that $\left| \int_C \frac{z+4}{z^3-1} dz \right| \leq \frac{6\pi}{7}$.

18. State and prove Cauchy's inequality.

19. *Give two Laurent series expansions in powers of z for the function* $f(z) = \frac{1}{z^2(1-z)}$ *and specify the regions in which those expansions are valid.*

20. State and prove Cauchy's Residue Theorem.

21. Define the essential singular points of a complex function with example. Verify the example with its series representations

(6×5=30)

Part C

Answer any ***two*** *questions.*

Each question carries ***15*** *marks.*

22. a) Show that $f(z)=ze^z$ is analytic and find its derivative
    b) Verify CR equations for the function f given by $f(z)=\frac{x-iy}{x^2+y^2}$

23.
- State and prove Fundamental theorem of Algebra.
- State and prove Maximum modulus principle.

24. State and prove Taylor's Theorem and hence deduce Maclaurin's theorem.

25. State and prove a necessary and sufficient condition for an isolated singular point z_0 of a function $f(z)$ to be a pole of order m. Derive the formula for residue at z_0 of $f(z)$. Find the residue at $z = i$ of $f(z) = \frac{(Logz)^3}{z^2+1}$.

(2×15=30)